RADIATIVE FIELDS IN SPACETIMES WITH MINKOWSKI AND DE SITTER ASYMPTOTICS*

JIŘÍ BIČÁK AND PAVEL KRTOUŠ

Institute of Theoretical Physics,
Faculty of Mathematics and Physics, Charles University,
V Holešovičkách 2, 180 00 Prague 8, Czech Republic

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The classical Bondi-Penrose approach to the gravitational radiation theory in asymptotically flat spacetimes is recalled and recent advances in the proofs of the existence of such spacetimes are briefly reviewed. We then mention the unique role of the boost-rotation symmetric spacetimes, representing uniformly accelerated objects, as the only explicit radiative solutions known which are asymptotically flat; they are used as test beds in numerical relativity and approximation methods.
The main part of the review is devoted to the examples of radiative fields in the vacuum spacetimes with positive cosmological constant. Type *N* solutions are analyzed by using the equation of geodesic deviation. Both these and Robinson-Trautman solutions of type *II* are shown to approach de Sitter universe asymptotically. Recent work on the the radiative fields due to uniformly accelerated charges in de Sitter spacetime ("cosmological Born's solutions") is reviewed and the properties of these fields are discussed with a perspective to characterize general features of radiative fields near a de Sitter-like infinity.

1. Introduction

After several important contributions to the gravitational radiation theory in the late 1950's and early 1960's by Pirani, Bondi, Robinson, Trautman and others, a landmark paper by Bondi et al[4] appeared in which radiative properties of isolated (spatially bounded) axisymmetric systems were studied along outgoing null hypersurfaces $u =$ constant, with u representing a retarded time function. An ansatz was made that the asymptotically Minkowski metric along $u =$ constant can be expanded in inverse powers

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of r,

$$g_{\mu v} = \eta_{\mu v} + h_{\mu v}(\theta) r^{-1} + f_{\mu v}(\theta) r^{-2} + \ldots, \tag{1}$$

where r denotes a suitable parameter along null generators (parametrized by coordinates θ, φ) on the hypersurfaces $u =$ constant. Under the assumption (1), Einstein's vacuum equations with the vanishing cosmological constant Λ were shown to determine uniquely formal power series solution of the form (1), provided that a free "news function" $c(u, \theta)$ is specified. The news function contains all information about radiation at infinity ("$r = \infty$"). It enters the fundamental "Bondi mass-loss formula" for the total mass $M(u)$ of an isolated system at retarded time u. Field equations imply that $M(u)$ is a monotonically decreasing function of u if $\partial_u c \neq 0$. A natural interpretation is that gravitational waves carry away positive energy from the system and thus decrease its mass. In the work of Bondi et al[4] as well as in the important generalizations by Sachs, Newman and Penrose (see, e.g., Ref. 5), the decay of radiative fields was studied in preferred coordinate systems.

In 1963 Penrose[6] formulated a beautiful *geometrical* framework for description of the "radiation zone" in general relativity in terms of conformal infinity.

Penrose's definition of asymptotically flat radiative spacetimes avoids such problems as "distances" or "suitable coordinates", and incorporates a clear definition of what is infinity. It is inspired by the work on radiation theory mentioned above, and by the properties of conformal infinity in Minkowski spacetime. In contrast to an Euclidean space, one can go to infinity in various directions: moving along timelike geodesics we come to the future (or past) timelike infinity I^+ (or I^-); along null geodesics (cf. Eq. (1)) we reach the future (past) null infinity $\mathcal{I}^+ (\mathcal{I}^-)$; spacelike geodesics lead to spatial infinity i_0. An asymptotically flat spacetime can be compactified and mapped into a finite region by an appropriate conformal transformation. Thus one obtains the well-known Penrose diagram in which the three types of infinities are mapped into the boundaries of the compactified spacetime.

It is generally accepted that Penrose's definition forms the only rigorous, geometrical basis for the discussion of gravitational radiation from isolated systems. It enables us to use techniques of local geometry "at infinity". For example, in the classical papers by Bondi et al[4,5], the decay of the curvature along outgoing null geodesics at infinity exhibits "the peeling-off" properties: the fall-off of various components of the Weyl tensor is

related to their Petrov algebraic type. To be more specific, certain complex linear combinations of the Weyl tensor in the orthonormal frame, $\Psi_k(k = 0,1,2,3,4)$, behave as $\Psi_k = O(r^{k-5})$ as $r \to \infty$. (In particular, $\Psi_4 \sim r^{-1}$ has the same algebraic structure as the Weyl tensor of a plane wave – radiative field of a bounded system resembles asymptotically that of a plane wave.) This decay of the curvature can be shown to follow (see e.g.[7]) from a sufficient differentiability (smoothness) of the conformally rescaled (unphysical) metric $\tilde{g}$ at the boundary representing null infinity.

In the next section we shall review some delicate rigorous mathematical results proving the existence of asymptotically flat vacuum spacetimes. We shall see that although such spacetimes satisfying the peeling properties have recently been shown to exist, it is far from clear what is their status in completely generic situations, in particular when sources are present. The boost-rotation symmetric spacetimes, representing uniformly accelerated "particles" or black holes, the role of which will be briefly discussed in Sec. 4, do exhibit the peeling properties,[8] but they contain two Killing vectors, and although they admit global smooth null infinity, this is singular at the points where particles "start" and "end".

Surprisingly perhaps, the existence of generic spacetimes was proved more then 15 years ago in the case of vacuum spacetimes with a positive cosmological constant[9,10] (Sec. 3). However, until now a little attention, within the framework of exact theory, has been paid to gaining a more "physical" picture of radiation propagating in spacetimes which are not asymptotically flat. In Secs. 5 and 6 we discuss exact solutions of Petrov types N and II of the vacuum field equations with $\Lambda > 0$ and interpret them as waves which at large times — near future infinity which is in this case spacelike — decay and leave "bald" de Sitter spacetime (cosmic no-hair). Finally, in Sec. 7, which is more technical and detailed than preceeding sections, we analyze the scalar and electromagnetic test fields from uniformly accelerated charges in de Sitter spacetime. And in the concluding "Outlook", we comment on more general cases of (non-test) fields near a spacelike infinity of de Sitter type.

It is natural to use de Sitter space for studying radiating sources in spacetimes which are not asymptotically flat and possess disjoint future and past infinities which are spacelike. It is the space of constant curvature, conformal to Minkowski space, and with Huygens principle satisfied for conformally invariant fields. The de Sitter universe also plays an important role in cosmology — not only in the context of inflationary theories but also as the "asymptotic state" of standard cosmological models with $\Lambda > 0$,

which has been indeed suggested by recent observations.

2. Asymptotically flat radiative spacetimes: existence

Despite its rigour and elegance, Penrose's definition of asymptotically flat spacetimes might turn to be of a limited importance if no interesting radiative spacetimes *exist* which satisfy the definition. In Section 4 we shall describe special exact radiative spacetimes which represent "uniformly accelerated" sources in general relativity and admit $\mathcal{I}$ as required; however, at least four points on $\mathcal{I}$ - those in which worldlines of the sources start and end - are singular. There are no other *explicit* exact radiative solutions describing finite sources available at present and this situation will probably not change soon. Nevertheless, thanks to the work of Friedrich[10], Christodoulou and Klainerman[11], and most recently due to works of Corvino and Schoen[12,13] and Chruściel and Delay[14] we know that globally non-singular (including *all* $\mathcal{I}$) asymptotically flat exact vacuum solutions of Einstein's equations really exist.

The key idea of Friedrich is in realizing that Penrose's treatment of infinity not only permits to use the methods of local differential geometry at $\mathcal{I}$ but also to analyze global existence problems of solutions of Einstein's equations in the physical spacetime by solving initial value problems in the conformally related unphysical compact spacetime. By using this approach Friedrich established that formal Bondi-type expansions (1) converge locally at $\mathcal{I}^+$. He succeeded to show that one can formulate the "hyperboloidal initial value problem" for Einstein's vacuum equations in which initial data are given on a hyperboloidal spacelike hypersurface $\mathcal{H}$ which intersects $\mathcal{I}^+$. It can then be proven that hyperboloidal initial data, which are sufficiently close to Minkowskian hyperboloidal data (i.e. to the metric induced on the hypersurface $\mathcal{H}$ in Minkowski spacetime by the standard Minkowski metric), evolve to a vacuum spacetime which is smooth on $\mathcal{I}^+$ and I^+ as required by Penrose's definition.

However, in a "complete picture" we would like to have initial data given on a *standard* spacelike Cauchy hypersurface which does not intersect $\mathcal{I}^+$ but "ends" at spatial infinity, rather than data given on a hyperboloidal initial hypersurface. A remarkable progress in proving rigorously the existence of general, asymptotically flat radiative spacetimes was achieved by Christodoulou and Klainerman. Their treatise[11] contains the first really global general existence statement for full, nonlinear Einstein's vacuum equations with vanishing cosmological constant: Any smooth asymptoti-

cally flat initial data set (determined by the first and the second fundamental form on a Cauchy hypersurface) which is "near flat (Minkowski) data" leads to a unique, smooth and geodesic complete development solution of Einstein's vacuum equations. This solution is "globally asymptotically flat" in the sense that the curvature tensor decays to zero at infinity in all directions.

The work of Friedrich, Christodoulou, Klainerman and others demonstrates rigorously that the general picture of null infinity is compatible with the vacuum Einstein field equations. However, important open questions remain. As noted in Introduction, the decay of the curvature (characterized by the Weyl tensor) along outgoing null geodesics at infinity exhibits the peeling properties. The results of Christodoulou and Klainerman, however, show a weaker peeling. They were only able to prove that the asymptotically flat vacuum initial data lead to $\Psi_0 \sim r^{-\frac{7}{2}}$ (*not* $\sim r^{-5}$) at null infinity.

That *particular* radiative spacetimes exist which are "asymptotically simple", i.e., they satisfy all Penrose's requirements, was proved by Cutler and Wald[15]. They show how to construct data on a standard Cauchy hypersurface $\mathcal{C}$ such that the data are near to flat spacetime data and coincide exactly with standard vacuum Schwarzschild data outside of a compact region. Hence, *on* initial hypersurface, there is a spherically symmetric vacuum gravitational field outside the compact region. As a consequence the future maximal evolution of the data is large enough to contain a Friedrich's hyperboloidal hypersurface $\mathcal{H}$ which terminates on $\mathcal{I}^+$. On this hypersurface the data evolved from the original data will still be close to flat spacetime data, i.e., to Minkowskian hyperboloidal initial data. By involving Friedrich's theorem we then know that the spacetime will be smoothly asymptotically flat in the future of this hypersurface. In fact, Cutler and Wald did not succeed in constructing the solutions within the Einstein theory in vacuum but, rather, within the Einstein-Maxwell theory.

Recently, however, Corvino and Schoen[12] have shown how vacuum Schwarzschild initial data can be glued to asymptotically flat time-symmetric vacuum data in compact region. (For the most recent generalization to the Kerr data see Ref. 13.) Starting from their construction Chruściel and Delay[14] obtained vacuum initial data which, using Friedrich's results, lead to asymptotically simple radiative spacetimes. Hence, one knows at present how to produce an infinite dimensional family of vacuum, asymptotically flat radiative spacetimes which exhibit the peeling properties and yield smooth $\mathcal{I}$.

In a completely general situation one may still encounter so-called

polyhomogeneous[16] $\mathcal{I}$, rather than smooth $\mathcal{I}$. The metric is called polyhomogeneous if at large r it admits an expansion in terms of $r^{-j}\log^i r$ rather than r^{-j} (as it has been assumed in the works of Bondi and others - cf. Eq. (1)). The hypothesis of polyhomogeneity of $\mathcal{I}$ has been shown to be formally consistent with Einstein's vacuum equations, the Bondi mass-loss law can be formulated, and the peeling properties of the curvature hold, with the first two terms identical to the standard peeling, the third term being $\sim r^{-3}\log r$.

In his more recent investigations[10], Friedrich constructed the new - finite but "wider" than the point i_0 - representation of spacelike infinity which enables one to make much deeper analysis of the initial data in the region where null infinity touches spacelike infinity. Good chances now exist to obtain clear criteria determining which data lead to the smooth and which just to the polyhomogeneous null infinity. First results in this direction have been obtained for the linearized spin-2 equations[17].

The ultimate goal of rigorous work on the existence and asymptotics of solutions of the Einstein equations is *physics*: one hopes to be able to consider (astro)physical *sources*, to relate their behaviour to the characteristics of the far fields. One would like to have under control various (both analytical and numerical) approximation procedures. A still more ambitious program is to consider strong initial data so as to be able to analyze such issues as cosmic censorship.

3. Asymptotically de Sitter radiative spacetimes: existence

Curiously enough, in the case of vacuum Einstein's equations with a *non-vanishing cosmological constant* Λ a more complete picture is known for some time already. By using his regular conformal field equations, Friedrich demonstrated[9] that initial data sufficiently close to de-Sitter data develop into solutions of Einstein's equations with a positive cosmological constant, which are asymptotically simple (with a smooth conformal infinity), as required in the original framework of Penrose. In the case $\Lambda > 0$ the infinity splits into two disjoint parts $\mathcal{I}^+$ and $\mathcal{I}^-$ which are both *spacelike*. Friedrich's theorem[9,10] proves "nonlinear stability of asymptotic simplicity of de Sitter space": Changing the de Sitter Cauchy data by a finite, but sufficiently small amount, one gets again a solution satisfying the definition of asymptotic simplicity for $\Lambda > 0$. This involves completeness of null geodesics, behavior of the conformal factor and metrics[9,10]. However, the directional dependence of radiative fields near spacelike infinities has not

been studied so far. This point will be discussed in Secs. 7 and 8.

Later Friedrich[18] also analyzed the existence of asymptotically simple solutions to the Einstein vacuum equations with a negative cosmological constant.

4. Asymptotically flat radiative spacetimes with boost-rotation symmetry

These spacetimes representing "uniformly accelerated objects" (see Fig. 1) have been reviewed in various places (see e.g.[1,2,3,19] and references therein); here we shall just mention few new developments.

The unique role of the boost-rotation symmetric spacetimes is exhibited by a theorem (see Ref. 20 and references therein) which roughly states that in axially symmetric, locally asymptotically flat electrovacuum spacetimes (in the sense that a null infinity satisfying Penrose's requirements exists, but it need not necessarily exist globally), the only additional symmetry that does not exclude radiation is the *boost* symmetry.

In Ref. 20 the general functional forms of the news functions (both gravitational and electromagnetic), and of the mass aspect and total Bondi mass of boost-rotation symmetric spacetimes are given (see also Ref. 8). Recently similar results were obtained[21] by using the Newman-Penrose formalism

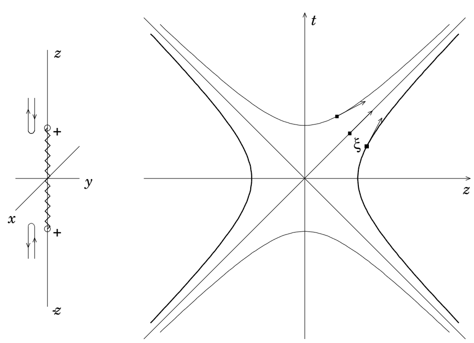

Figure 1. Two particles uniformly accelerated in opposite directions. The orbits of the boost Killing vector (thinner hyperbolas) are spacelike in the region $t^2 > z^2$. Here, the boost-rotational symmetric metrics can *locally* be transformed into the metrics of Einstein-Rosen cylindrical waves.

and under more general assumptions (for example, $\mathcal{I}$ could in principle be polyhomogeneous).

The general structure of the boost-rotation symmetric spacetimes with hypersurface orthogonal Killing vectors was analyzed in detail in[22]. Their radiative properties, including explicit construction of radiation patterns and of Bondi mass for the specific boost-rotation symmetric solutions were investigated in several works – we refer to the reviews[1,2,8,19] for details. Recently the Newtonian limit of these spacetimes was analyzed[23]. The boost-rotation symmetric spacetimes have also played the role in such diverse fields like numerical relativity and quantum production of black-hole pairs[39]. For the most recent use of the boost-rotation symmetric solutions as test beds in numerical relativity, see Ref. 24, where Friedrich's conformal field equations are solved numerically with specific boost-rotation symmetric solutions being employed to check the code.

5. On physical interpretation of non-twisting type *N* solutions with $\Lambda \neq 0$

In order to gain an intuitive picture of radiative spacetimes with $\Lambda \neq 0$, we considered[25,26] all non-twisting Petrov-type N solutions of vacuum Einstein's field equations with cosmological constant. They belong either to the non-expanding Kundt class $KN(\Lambda)$ or to the expanding Robinson-Trautman class $RTN(\Lambda)$. We defined invariant subclasses of each class and gave the corresponding metrics explicitly in suitable canonical coordinates and interpreted them by analyzing the equation of geodesic deviation.

In the case of the Kundt class, a suitable coordinate system $(v, \xi, \bar{\xi}, u)$, where $\xi, \bar{\xi}$ are space-like coordinates, v is a parameter along the null geodesics tangent to the null vector $\mathbf{k}$ and u is a retarded time with $u =$ constant being a wavefront, can be introduced in which the $KN(\Lambda)$ metrics have the form

$$ds^2 = 2\frac{1}{p^2}d\xi d\bar{\xi} - 2\frac{q^2}{p^2}du dv + F du^2 \ , \tag{2}$$

where

$$p = 1 + \frac{\Lambda}{6}\xi\bar{\xi} \ , \qquad q = (1 - \frac{\Lambda}{6}\xi\bar{\xi})\alpha + \bar{\beta}\xi + \beta\bar{\xi} \ ,$$
$$F = \kappa\frac{q^2}{p^2}v^2 - \frac{(q^2)_{,u}}{p^2}v - \frac{q}{p}H \ , \qquad \kappa = \frac{\Lambda}{3}\alpha^2 + 2\beta\bar{\beta} \ .$$

Here $\alpha(u)$ and $\beta(u)$ are *arbitrary* real and complex functions of u, respectively. These functions play the role of two arbitrary 'parameters',

i.e., we can denote the Kundt class by $KN(\Lambda) \equiv KN(\Lambda)[\alpha,\beta]$. The function $H = H(\xi,\bar{\xi},u)$ entering F is restricted by Einstein's equations, $H_{,\xi\bar{\xi}} + (\Lambda/3p^2)\,H = 0$. There exists a general solution to this equation

$$H(\xi,\bar{\xi},u) = (f_{,\xi} + \bar{f}_{,\bar{\xi}}) - \frac{\Lambda}{3}p(\bar{\xi}f + \xi\bar{f}) \ , \qquad (3)$$

where $f(\xi,u)$ is an arbitrary function of ξ and u, analytic in ξ. The spacetime is conformally flat if and only if the structural function H is of the form

$$H = H_c = \frac{1}{p}\Big[(1 - \frac{\Lambda}{6}\xi\bar{\xi})\mathcal{A} + \bar{\mathcal{B}}\xi + \mathcal{B}\bar{\xi}\Big] \ , \qquad (4)$$

with $\mathcal{A}(u)$ and $\mathcal{B}(u)$ being arbitrary real and complex functions, respectively. Since H_c of this form corresponds to (3) for f quadratic in ξ we easily infer that the $KN(\Lambda)$ solutions (2), (3) with $f = f_c = c_0(u) + c_1(u)\xi + c_2(u)\xi^2$, where $c_i(u)$ are arbitrary complex functions of u, are isometric to Minkowski (if $\Lambda = 0$), de Sitter ($\Lambda > 0$) and anti-de Sitter ($\Lambda < 0$) spacetime.

The Robinson-Trautman solutions[27] satisfying the vacuum equations with Λ can be written as[28]

$$ds^2 = 2\frac{r^2}{P^2}d\zeta d\bar{\zeta} - 2dudr - \Big[\Delta \ln P - 2r(\ln P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3}r^2\Big]du^2 \ , \qquad (5)$$

where ζ is a complex spatial coordinate, r is an affine parameter along the rays generated by the null vector field $\mathbf{k}$, u is a retarded time, m is a function of u which in some cases can be interpreted as mass, and $\Delta \equiv 2P^2\partial^2/\partial\zeta\partial\bar{\zeta}$. The function $P \equiv P(\zeta,\bar{\zeta},u)$ satisfies the Robinson-Trautman equation $\Delta\Delta(\ln P) + 12m(\ln P)_{,u} - 4m_{,u} = 0$. In this section we restrict attention to the solutions of type N and denote these as $RTN(\Lambda)$. In this case $m = 0$ and $\Delta \ln P = K(u)$. By a transformation $u = g(\tilde{u})$, $r = \tilde{r}/\dot{g}$, where $\dot{g} = dg/d\tilde{u}$, we can set the Gaussian curvature $K(u)$ of the 2-surfaces $2P^{-2}d\zeta d\bar{\zeta}$ to be $K = 2\epsilon$, where $\epsilon = +1, 0, -1$. Thus, the different subclasses can be denoted as $RTN(\Lambda,\epsilon)$. The corresponding metrics can be written as

$$ds^2 = 2\frac{r^2}{P^2}d\zeta d\bar{\zeta} - 2dudr - 2\Big[\epsilon - r(\ln P)_{,u} - \frac{\Lambda}{6}r^2\Big]du^2 \ . \qquad (6)$$

Since $\epsilon = +1, 0, -1$ and $\Lambda > 0$, $\Lambda = 0$, $\Lambda < 0$, 9 invariant subclasses exist.

Another coordinate system for the $RTN(\Lambda,\epsilon)$ class has been given. The metric is expressed in terms of a function $f(\xi,u)$ which is an arbitrary function of u, analytic in spatial coordinate ξ:

$$ds^2 = 2v^2 d\xi d\bar{\xi} + 2v\bar{\mathrm{A}}d\xi du + 2v\mathrm{A}d\bar{\xi}du + 2\psi dudv + 2(\mathrm{A}\bar{\mathrm{A}} + \psi\mathrm{B})du^2 \ , \qquad (7)$$

where

$$\mathrm{A} = \epsilon\xi - vf \ , \qquad \mathrm{B} = -\epsilon + \frac{v}{2}(f_{,\xi} + \bar{f}_{,\bar{\xi}}) + \frac{\Lambda}{6}v^2\psi \ , \qquad \psi = 1 + \epsilon\xi\bar{\xi} \ .$$

The non-vanishing Weyl tensor components are proportional to $f_{,\xi\xi\xi}$ so that the solutions are conformally flat if f is quadratic in ξ. Thus, the $RTN(\Lambda, \epsilon)$ solutions (7) with $f = f_c = c_0(u) + c_1(u)\xi + c_2(u)\xi^2$, where $c_i(u)$ are arbitrary complex functions of u, are isometric to Minkowski (if $\Lambda = 0$), de Sitter ($\Lambda > 0$) and anti-de Sitter ($\Lambda < 0$) spacetime.

It is natural to base the local characterization of these spacetimes on the equation of geodesic deviation

$$\frac{D^2 Z^\mu}{d\tau^2} = -R^\mu{}_{\alpha\beta\gamma} u^\alpha Z^\beta u^\gamma \ , \tag{8}$$

where $\mathbf{u} = d\mathbf{x}/d\tau$, $\mathbf{u}\cdot\mathbf{u} = -1$, is the four-velocity of a free test particle (observer), τ is the proper time, and $\mathbf{Z}(\tau)$ is the displacement vector. In order to obtain invariant results, one sets up a frame $\{\mathbf{e}_{(a)}\}$ along the geodesic. Choosing $\mathbf{e}_{(0)} = \mathbf{u}$ and perpendicular space-like unit vectors $\{\mathbf{e}_{(1)}, \mathbf{e}_{(2)}, \mathbf{e}_{(3)}\}$ in the local hypersurface orthogonal to $\mathbf{u}$ and by projecting (8) onto the frame, we get

$$\ddot{Z}^{(i)} = -R^{(i)}_{\ \ (0)(j)(0)} Z^{(j)} \ , \tag{9}$$

where $Z^{(j)} = \mathbf{e}^{(j)}\cdot\mathbf{Z} = e^{(j)}_\mu Z^\mu$ determine directly the distance between close test particles,

$$\ddot{Z}^{(i)} \equiv \mathbf{e}^{(i)} \cdot \frac{D^2\mathbf{Z}}{d\tau^2} = e^{(i)}_\mu \frac{D^2 Z^\mu}{d\tau^2} \tag{10}$$

are physical relative accelerations, and $R_{(i)(0)(j)(0)} = e^\alpha_{(i)} u^\beta e^\gamma_{(j)} u^\delta R_{\alpha\beta\gamma\delta}$. Setting $Z^{(0)} = 0$, all test particles are "synchronized" by τ. In the $KN(\Lambda)$ and $RTN(\Lambda, \epsilon)$ spacetimes the equations of geodesic deviation take the form

$$\begin{aligned} \ddot{Z}^{(1)} &= \frac{\Lambda}{3} Z^{(1)} - \mathcal{A}_+ Z^{(1)} + \mathcal{A}_\times Z^{(2)} \ , \\ \ddot{Z}^{(2)} &= \frac{\Lambda}{3} Z^{(2)} + \mathcal{A}_+ Z^{(2)} + \mathcal{A}_\times Z^{(1)} \ , \\ \ddot{Z}^{(3)} &= \frac{\Lambda}{3} Z^{(3)} \ , \end{aligned} \tag{11}$$

where the amplitudes of the (exact) gravitational waves are given by

$$\mathcal{A}_+(\tau) = \frac{1}{2} pq\dot{u}^2\, \mathcal{R}e\,\{f_{,\xi\xi\xi}\} \ , \qquad \mathcal{A}_\times(\tau) = \frac{1}{2} pq\dot{u}^2\, \mathcal{I}m\,\{f_{,\xi\xi\xi}\} \ , \tag{12}$$

for the $KN(\Lambda)$ spacetimes, and by

$$\mathcal{A}_+(\tau) = -\frac{1}{2}\frac{\psi}{v}\dot{u}^2\,\mathcal{R}e\,\{f_{,\xi\xi\xi}\}\ , \qquad \mathcal{A}_\times(\tau) = -\frac{1}{2}\frac{\psi}{v}\dot{u}^2\,\mathcal{I}m\,\{f_{,\xi\xi\xi}\}\ , \quad (13)$$

in the $RTN(\Lambda,\epsilon)$ spacetimes. Equations (11)-(13) give relative accelerations of the free test particles in terms of their actual positions. They enable us to draw a number of simple conclusions:

(1) All particles move isotropically one with respect to the other if no gravitational wave is present, i.e., if $f_{,\xi\xi\xi}=0$. In this case both the $KN(\Lambda)$ and $RTN(\Lambda,\epsilon)$ spacetimes are vacuum conformally flat, and therefore Minkowski ($\Lambda=0$), de Sitter ($\Lambda>0$) and anti-de Sitter ($\Lambda<0$). Such spaces are maximally symmetric, homogeneous, isotropic, and they represent a natural background for other "non-trivial" $KN(\Lambda)$ and $RTN(\Lambda,\epsilon)$ type N solutions.

(2) If amplitudes $\mathcal{A}_+$ and $\mathcal{A}_\times$ do not vanish ($f_{,\xi\xi\xi}\neq 0$), the particles are influenced by the wave in a similar way as they are affected by a standard gravitational wave on Minkowski background. However, if $\Lambda\neq 0$, the influence of the wave adds with the (anti-) de Sitter isotropic expansion (contraction). This makes plausible our interpretation of the $KN(\Lambda)$ and $RTN(\Lambda,\epsilon)$ metrics as *exact gravitational waves propagating on the constant curvature backgrounds.*

(3) The wave propagates in the space-like direction of $\mathbf{e}_{(3)}$ and has a *transverse character* since only motions in the perpendicular directions of $\mathbf{e}_{(1)}$ and $\mathbf{e}_{(2)}$ are affected. The propagation direction given by $\mathbf{e}_{(3)}$ coincides with the projection of the Debever-Penrose vector $\mathbf{k}$ on the hypersurface orthogonal to the observer's velocity $\mathbf{u}$.

(4) There are *two polarization modes* of the wave: "+" and "×", $\mathcal{A}_+$ and $\mathcal{A}_\times$ being the amplitudes. Under rotation in the transverse plane they transform in such a way so that the helicity of the wave is 2, as with linearized waves on Minkowski background.

(5) The waves have amplitude $\mathcal{A}=\frac{1}{2}pq\dot{u}^2\ |f_{,\xi\xi\xi}|$ for the $KN(\Lambda)$ class and $\mathcal{A}=\frac{1}{2}(\psi/v)\dot{u}^2\ |f_{,\xi\xi\xi}|$ for $RTN(\Lambda,\epsilon)$; this is invariant under rotations in the transverse plane.

For a *special* class of timelike geodesics characterized by $\xi=\xi_0=$ constant one can calculate the wave amplitudes explicitly in terms of τ for both classes. At large $\tau>0$ the amplitudes decay as $\mathcal{A}\sim\exp(-n\sqrt{\Lambda/3}\,\tau)$, $n>0$, i.e., *waves are damped exponentially.* The spacetimes approach asymptotically the de Sitter universe. This is an explicit demonstration of the *cosmic no-hair conjecture* (see, e.g., Ref. 29) under the presence of

waves within exact model spacetimes. In the following section we turn to the cosmic no-hair conjecture in the Robinson-Trautman spacetimes of Petrov type *II*.

Here, let us remark yet that a suitable physical interpretation of exact radiative spacetimes which are not asymptotically flat has been achieved by using the equation of geodesic deviation also in other cases. For example, Siklos solutions have been shown to represent exact gravitational waves in the anti-de Sitter universe.[30]

6. Robinson–Trautman radiative spacetimes with $\Lambda \neq 0$

Robinson-Trautman metrics are the general radiative vacuum solutions which admit a geodesic, shearfree and twistfree null congruence of diverging rays. In the standard coordinates the metric has the form given by Eq. (5), where $P \equiv P(u, \zeta, \bar{\zeta})$ satisfies the Robinson-Trautman fourth-order parabolic equation $\Delta\Delta(lnP) + 12m(lnP)_{,u} - 4m_{,u} = 0$. In the previous section we considered the Robinson-Trautman metrics of Petrov type N when the mass function $m = 0$. However, the best candidates for describing radiation from isolated sources are the Robinson-Trautman metrics of type II with the 2-surfaces S^2 (having spherical topology) given by $u, r =$ constant and $m \neq 0$. The Gaussian curvature of S^2 can be expressed as $K = \Delta lnP$. If $K =$ constant, we obtain the Schwarzschild solution with mass $m = K^{-\frac{3}{2}}$.

In the studies of the Robinson-Trautman spacetimes of Petrov type II with $\Lambda = 0$ it was shown that these spacetimes exist globally for all positive "times" and converge asymptotically to a Schwarzschild metric (see Ref. 31 and references therein). Interestingly, the extension of these spacetimes across the "Schwarzschild-like" event horizon can only be made with a finite degree of smoothness. All these rigorous studies are based on the derivation and analysis of an asymptotic expansion describing the long-time behavior of the solutions of the nonlinear parabolic Robinson-Trautman equation given above.

We analyzed Robinson-Trautman radiative spacetimes with the positive cosmological constant Λ in detail.[32,33] The results proving the global existence and convergence of the solutions of the Robinson-Trautman equation can be taken over from the previous studies since Λ does not explicitly enter this equation. We have shown that, starting with arbitrary, smooth initial data at $u = u_0$ (see Fig. 2), the cosmological Robinson-Trautman solutions converge exponentially fast to a Schwarzschild-de Sitter solution at large retarded times ($u \to \infty$). The interior of a Schwarzschild-de Sitter black hole

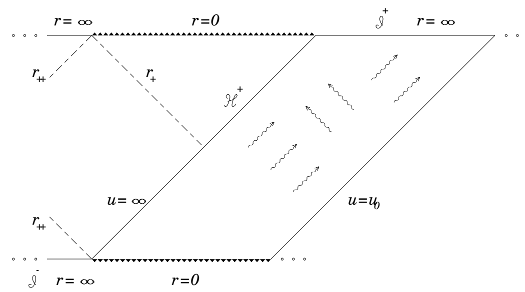

Figure 2. The evolution of the cosmological Robinson-Trautman solutions with a positive cosmological constant. A black hole with the horizon $\mathcal{H}^+$ is formed; at future infinity $\mathcal{I}^+$ the spacetime approaches a de Sitter spacetime exponentially fast, in accordance with the cosmic no-hair conjecture.

can be joined to an "external" cosmological Robinson-Trautman spacetime across the horizon $\mathcal{H}^+$. In some cases this joining can be made with a higher degree of smoothness than in the corresponding case with $\Lambda = 0$. We have also demonstrated that the cosmological Robinson-Trautman solutions represent explicit models exhibiting the cosmic no-hair conjecture: a geodesic observer outside of the black-hole horizon will see, inside his past light cone, these spacetimes to approach the de Sitter spacetime exponentially fast as he is approaching future (spacelike) infinity $\mathcal{I}^+$. For a freely falling observer the observable universe thus becomes quite bald. This is what the cosmic no-hair conjecture claims. As far as we are aware, these models represent the only exact analytic demonstration of the cosmic no-hair conjecture under the presence of gravitational waves. They also appear to be the only exact examples of a black-hole formation in nonspherical spacetimes which are not asymptotically flat. Hopefully, these models may serve as tests of various approximation methods, and as test beds in numerical studies of more realistic situations in cosmology.

7. Fields of uniformly accelerated sources in de Sitter spacetime

The question of the electromagnetic field and associated radiation from uniformly accelerated charges has been one of the best known "perpetual problems" in classical physics from the beginning of the last century. In the pioneering work in 1909, Born gave the time-symmetric solution for the

field of two point particles with opposite charges, uniformly accelerated in opposite directions in Minkowski space (cf. Fig. 1). Even the December 2000 issue of Annals of Physics contains three papers[34] with numerous references on "electrodynamics of hyperbolically accelerated charges".

In general relativity, solutions of Einstein's equations, representing "uniformly accelerated particles or black holes", are the *only* explicitly known exact *radiative* spacetimes describing *finite* sources. They were briefly described in Sec. 4.

Here, we present the summary of our recent work[35] in which we generalized the Born solutions for scalar and electromagnetic fields to the case of two charges uniformly accelerated in de Sitter universe and explicitly have shown how in the limit $\Lambda \to 0$ the Born solutions are retrieved. We also studied the asymptotic expansions of the fields in the neighborhood of future infinity $\mathcal{I}^+$. Since in de Sitter spacetime conformal infinities $\mathcal{I}^\pm$ are spacelike, there exist particle and event horizons. It is known[36] that the radiation field is "less invariantly" defined when $\mathcal{I}^+$ is spacelike (it depends on the direction in which $\mathcal{I}^+$ is approached), but no explicit model appears to be available so far. Our solutions can serve as prototypes for studying these issues.

In another work[37] we analyzed fields of accelerated sources to show the *insufficiency of purely retarded fields in de Sitter spacetime*. Consider a point P near $\mathcal{I}^-$ whose past null cone will not cross the particles' worldlines (Fig. 3). The field at P should vanish if an incoming field is absent. However, the "Coulomb-type" field of particles cannot vanish there because of Gauss law[38]. The requirement that the field be purely retarded leads in general to a bad behavior of the field along the "creation light cone" of the "point" at which a source enters the universe (see Ref. 37 for a detailed discussion).

The de Sitter universe has topology $S^3 \times \mathbb{R}$. The metric in standard "spherical" coordinates is

$$g_{\rm dS} = -\mathrm{d}\tau^2 + \alpha^2 \cosh^2(\tau/\alpha) \left(\mathrm{d}\chi^2 + \sin^2\chi \, \mathrm{d}\omega^2\right) , \qquad (14)$$

where $\mathrm{d}\omega^2 = \mathrm{d}\vartheta^2 + \sin^2\!\vartheta \, \mathrm{d}\varphi^2$, $\tau \in \mathbb{R}$, and $\alpha^2 = 3/\Lambda$. Putting $\chi = \tilde{r}$, $\tau = \alpha \log\tan(\tilde{t}/2)$, $\tilde{t} \in \langle 0, \pi\rangle$, in Eq. (14), the de Sitter metric can be written in the form

$$g_{\rm dS} = \alpha^2 \sin^{-2}\tilde{t} \left(-\mathrm{d}\tilde{t}^{\,2} + \mathrm{d}\tilde{r}^2 + \sin^2\!\tilde{r} \, \mathrm{d}\omega^2\right) . \qquad (15)$$

The lines $\tilde{r} = \pi$ and $\tilde{r} = -\pi$ are identified, the spacelike hypersurfaces $\tilde{t} = 0, \pi$ represent $\mathcal{I}^-$ and $\mathcal{I}^+$ (Fig. 3).

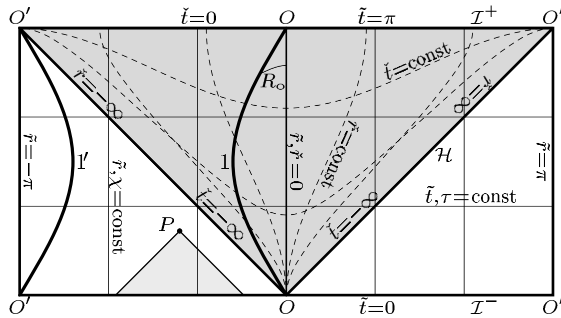

Figure 3. The conformal diagram of de Sitter spacetime. Uniformly accelerated particles move along worldlines 1 and $1'$. The shaded region is the domain of influence of 1, its boundary $\mathcal{H}$ is the "creation light cone" of this particle "born" at $\tilde{t} = 0$ at "point" O. Retarded fields of 1 and $1'$ cannot affect point P, a Coulomb-type field, however, cannot vanish there.

We recently studied[35,37] two particles moving with uniform acceleration in de Sitter space. Their worldlines are plotted in Fig. 3 as 1, $1'$ (for explicit formulae see Ref. 37, Eq. (4.4)). Both particles start at antipodes of the spatial section of de Sitter space at $\mathcal{I}^-$ and move one towards the other until $\tilde{t} = \pi/2$, the moment of the maximal contraction of de Sitter space. Then they move, in a time-symmetric manner, apart from each other until they reach future infinity at the antipodes from which they started. Their physical velocities, as measured in the "comoving" coordinates $\{\tau, \chi, \vartheta, \varphi\}$, have simple form $v_\chi = \sqrt{g_{\chi\chi}}\, d\chi/d\tau = \mp a_\mathrm{o}\alpha \tanh(\tau/\alpha)\, [1 + a_\mathrm{o}^2\alpha^2 \tanh^2(\tau/\alpha)]^{-1/2}$, where $|a_\mathrm{o}|$ is the magnitude of their acceleration. In contrast to the flat space case, the particles do *not* approach the velocity of light in the "natural" global coordinate system. Nevertheless, they are causally disconnected (Fig. 3) as in the flat space case: no signal from one particle can reach the other particle.

Two charges moving along the orbits of the boost Killing vector in flat space are *at rest* in the Rindler coordinate system and have a constant distance from the spacetime origin, as measured along the slices orthogonal to the Killing vector. Similarly, the worldlines 1 and $1'$ are the orbits of the "static" Killing vector $\partial/\partial T$ of de Sitter space. In static coordinates $\{T, R, \vartheta, \varphi\}$, $T = \frac{\alpha}{2} \log[(\cos\tilde{r} - \cos\tilde{t})/(\cos\tilde{r} + \cos\tilde{t})]$, $R = \alpha \sin\tilde{r}/\sin\tilde{t}$, the particles 1, $1'$ are at rest at $R = \pm R_\mathrm{o} = \mp a_\mathrm{o}\alpha^2/\sqrt{1 + a_\mathrm{o}^2\alpha^2}$, with four-accelerations $-(R_\mathrm{o}/\alpha^2)\, \partial/\partial R$. The particle 1 ($1'$) has, as measured at fixed T, a constant proper distance from the origin $\tilde{t} = \pi/2$, $\tilde{r} = 0$ ($\tilde{r} = \pi$). As with Rindler coordinates in Minkowski space, the static coordinates cover

only a "half" of de Sitter space; in the other half the Killing vector $\partial/\partial T$ becomes spacelike.

By the conformal transformation of the boosted Coulomb fields in Minkowski space, we constructed [37] test scalar and electromagnetic fields produced by charges moving along the worldlines 1, 1′ in de Sitter space. The scalar field from two *identical* scalar charges s is given by

$$\Phi_{\rm sym} = (s/4\pi)\ \mathcal{Q}^{-1}\ , \tag{16}$$

$$\mathcal{Q} = \left[\alpha^2\big(\sqrt{1+a_{\rm o}^2\alpha^2} + a_{\rm o}R\cos\vartheta\big)^2 - \alpha^2 + R^2\right]^{\frac{1}{2}} \tag{17}$$

(Ref. 37, Eq. (5.4)), whereas the electromagnetic field due to *opposite* charges $+e$ and $-e$ is (Ref. 37, Eq. (5.7))

$$\begin{aligned} F_{\rm sym} = -\frac{e}{4\pi}\,\frac{1}{\mathcal{Q}^3}\frac{a_{\rm o}\,\alpha^4}{\sin^3\tilde{t}}\,\Big[&\cos\tilde{t}\sin^2\tilde{r}\sin\vartheta\,{\rm d}\tilde{r}\wedge{\rm d}\vartheta \\ &+ (a_{\rm o}^{-1}\sqrt{a_{\rm o}^2+\alpha^{-2}}\sin\tilde{r} + \sin\tilde{t}\cos\vartheta)\,{\rm d}\tilde{t}\wedge{\rm d}\tilde{r} \\ &- \sin\tilde{t}\cos\tilde{r}\sin\tilde{r}\sin\vartheta\,{\rm d}\tilde{t}\wedge{\rm d}\vartheta\Big]\ . \end{aligned} \tag{18}$$

We call these smooth (outside the sources) fields symmetric because they

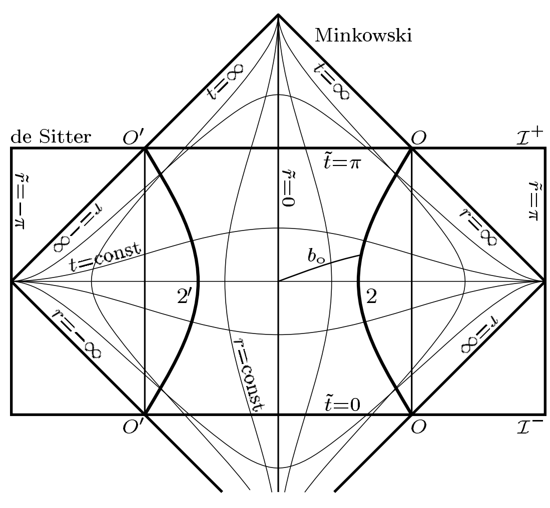

Figure 4. The worldlines 2, 2′ of uniformly accelerated charges symmetrically located with respect to the origins of both de Sitter and conformally related Minkowski spacetimes.

can be written as a symmetric combination of retarded and advanced effects from both charges.

Although Eqs. (16) and (18) represent fields due to uniformly accelerated charges in de Sitter space, their relation to the Born solutions is not transparent because the sources are not located symmetrically with respect to $\tilde{r} = 0$. Hence, we considered the worldlines 2 and 2′ (Fig. 4) which, due to homogeneity and isotropy of de Sitter space, also represent uniformly accelerated particles. These worldlines and the resulting fields can be obtained from Eqs. (16)–(18) by a spatial rotation by $\pi/2$. We find the worldlines 2, 2′ to be given by $\cot\tilde{t} = -\sinh\big(\lambda_{\rm dS}\alpha^{-1}\sqrt{1+a_{\rm o}^2\alpha^2}\big)/\sqrt{1+a_{\rm o}^2\alpha^2}$, $\tan\tilde{r} = \pm\cosh\big(\lambda_{\rm dS}\alpha^{-1}\sqrt{1+a_{\rm o}^2\alpha^2}\big)/(a_{\rm o}\alpha)$, $\vartheta = 0$, $\varphi = 0$. The scalar and electromagnetic fields are

$$\Phi_{\rm BdS} = (s/4\pi)\,\sin\tilde{t}\;(\sin\tilde{t} + \cos\tilde{r})^{-1}\;\mathcal{R}^{-1}\;, \tag{19}$$

$$\begin{aligned} F_{\rm BdS} = &-\frac{e}{4\pi}\frac{\alpha^3}{\mathcal{R}^3}\frac{a_{\rm o}\alpha\sin\vartheta}{(\sin\tilde{t}+\cos\tilde{r})^3}\big[\sin^2\tilde{r}\cos\tilde{t}\,{\rm d}\tilde{r}\wedge{\rm d}\vartheta \\ &-(a_{\rm o}^{-1}\sqrt{a_{\rm o}^2+\alpha^{-2}}\cos\tilde{r}-\sin\tilde{t})\,\cot\vartheta\,{\rm d}\tilde{t}\wedge{\rm d}\tilde{r} \\ &+(a_{\rm o}^{-1}\sqrt{a_{\rm o}^2+\alpha^{-2}}-\cos\tilde{r}\sin\tilde{t})\,\sin\tilde{r}\,{\rm d}\tilde{t}\wedge{\rm d}\vartheta\big]\;, \end{aligned} \tag{20}$$

$$\frac{\mathcal{R}}{\alpha} = \frac{\big[(a_{\rm o}\alpha\sin\tilde{t}-\sqrt{1+a_{\rm o}^2\alpha^2}\cos\tilde{r})^2+\sin^2\tilde{r}\sin^2\vartheta\big]^{\frac{1}{2}}}{\sin\tilde{t}+\cos\tilde{r}}\;.$$

In order to understand explicitly the relation of these fields to the classical Born solutions, we considered Minkowski spacetime with spherical coordinates $\{t, r, \vartheta, \varphi\}$ and set $t = -\alpha\cos\tilde{t}/(\cos\tilde{r}+\sin\tilde{t}\,)$, $r = \alpha\sin\tilde{r}/(\cos\tilde{r}+\sin\tilde{t}\,)$. In coordinates $\{t, r, \vartheta, \varphi\}$, which can also be used in de Sitter space, the worldlines 2, 2′ acquire the simple form: $\vartheta = 0$, $\varphi = 0$, and $t = b_{\rm o}\sinh(\lambda_{\rm M}/b_{\rm o})$, $r = \pm b_{\rm o}\cosh(\lambda_{\rm M}/b_{\rm o})$, where $\lambda_{\rm M}$ is the proper time as measured by Minkowski metric, and $b_{\rm o}/\alpha = \sqrt{1+a_{\rm o}^2\alpha^2} - a_{\rm o}\alpha$. These worldlines are just two hyperbolae (Fig. 4), representing particles with uniform acceleration $1/b_{\rm o}$ as measured in Minkowski space. Transforming the fields (19) and (20) into conformally flat coordinates $\{t, r, \vartheta, \varphi\}$, we obtain the fields in the form which in the limit $\Lambda \to 0$ yields easily the classical Born fields of uniformly accelerated charges in Minkowski space (see Ref. 35 for details).

What is the character of the generalized Born fields? Focusing on the electromagnetic case, we first decompose the field (20) into the orthonormal tetrad tied to coordinates $\{\tilde{t}, \tilde{r}, \vartheta, \varphi\}$. Splitting the field into the electric

and magnetic parts we get

$$
\begin{aligned}
E = {} & \frac{e}{4\pi}\frac{\alpha\sin^2\tilde{t}}{\mathcal{R}^3(\sin\tilde{t}+\cos\tilde{r})^3}\times \\
& \left[-\left(\sqrt{1+a_{\mathrm{o}}^2\alpha^2}\cos\tilde{r}-a_{\mathrm{o}}\alpha\sin\tilde{t}\right)\cos\vartheta\,\mathrm{e}_{\tilde{r}}\right. \\
& \left.+\left(\sqrt{1+a_{\mathrm{o}}^2\alpha^2}-a_{\mathrm{o}}\alpha\sin\tilde{t}\cos\tilde{r}\right)\sin\vartheta\,\mathrm{e}_{\vartheta}\right]\,, \\
B = {} & -\frac{e}{4\pi}\frac{a_{\mathrm{o}}\alpha^2\sin^2\tilde{t}}{\mathcal{R}^3(\sin\tilde{t}+\cos\tilde{r})^3}\cos\tilde{t}\sin\tilde{r}\sin\vartheta\,\mathrm{e}_{\varphi}\ .
\end{aligned}
\tag{21}
$$

The fields exhibit some features typical for the classical Born solution. The toroidal electric field, E_φ, vanishes, only B_φ is non-vanishing. At $\tilde{t}=\pi/2$, the moment of time symmetry, $B_\varphi = 0$. It vanishes also for $\vartheta = 0$ — there is no Poynting flux along the axis of symmetry.

The classical Born field decays rapidly ($E \sim r^{-4}$, $B \sim r^{-5}$) at spatial infinity, but it is "radiative" ($E, B \sim r^{-1}$) if we expand it along null geodesics $t - r =$ constant, approaching thus null infinity. (This behavior is typical also for boost-rotation symmetric spacetimes with $\Lambda = 0$ mentioned in Sec. 4.) In de Sitter spacetime with standard slicing, the space is finite (S^3). However, we can approach infinity along spacelike hypersurfaces if, for example, we consider the "steady-state" half of de Sitter universe (cf. Fig. 3) with flat-space slices, i.e., if we take the "conformally flat" time $\check{t} =$ constant; the tetrad components of the fields then decay as $\check{r}^{-2}$.

The fields decay very rapidly along *timelike* worldlines as $\mathcal{I}^+$ is approached. This is caused by the exponential expansion of slices $\tau =$ constant (cf. Eq. (14)). As $\tau \to \infty$ the electric field (21) becomes radial, $E_{\tilde{r}} \sim \exp(-2\tau/\alpha)$, and $B_\varphi \sim \exp(-2\tau/\alpha)$. The energy density, $u = \frac{1}{2}(E^2 + B^2)$, decays as (expansion factor)$^{-4}$ — as energy density in the radiation dominated standard cosmologies. This behavior corresponds to the "cosmic no-hair" properties in spacetimes with $\Lambda > 0$ discussed in Secs. 5 and 6.

To study the asymptotic behavior of a field along a null geodesic, we have to (i) find a geodesic and parameterize it by an affine parameter ζ, (ii) construct a tetrad parallelly propagated along the geodesic, and (iii) study the asymptotic expansion of the tetrad components of the field. We find that along null geodesics lying in the axis $\vartheta = 0$ (thus crossing the particles' worldlines) the "radiation field", i.e. the coefficient of the leading term in $1/\zeta$, vanishes, as could have been anticipated — particles do not radiate in the direction of their acceleration. The radiation field also vanishes along null geodesics reaching infinity along directions *opposite* to

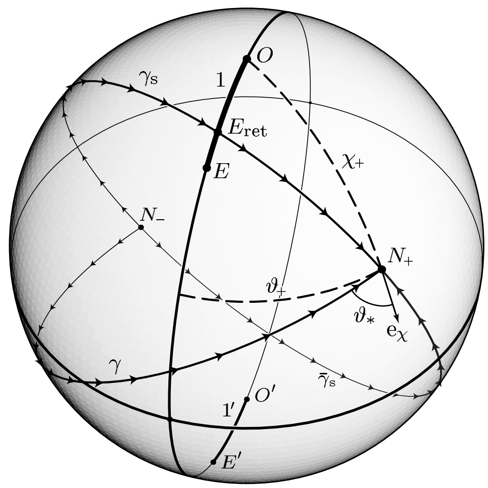

Figure 5. Space trajectories of null geodesics γ, γ_s and $\bar{\gamma}_s$ indicated on the slice $\tilde{t} = \text{constant}$ $(\varphi = 0)$. Charges 1, $1'$ move along $\vartheta = 0$ from poles O, O' to points E, E' and back. γ, γ_s and $\bar{\gamma}_s$ start at N_- at $\tilde{t} = 0$ and arrive at N_+ (with coordinates χ_+, ϑ_+) at $\tilde{t} = \pi$. The direction of γ at N_+ is specified by angles ϑ_*, φ_* (φ_* describes rotation around e_χ in the dimension not seen). γ_s crosses the worldline of particle 1 at E_{ret}, $\bar{\gamma}_s$ reaches N_+ from the opposite direction.

those of geodesics emanating from the particles (see Fig. 5). Along all other geodesics the field *has* radiative character. Along a null geodesic coming from a general direction to a general point on $\mathcal{I}^+$ we find the electric and magnetic fields (in a parallelly transported tetrad $\{f_\mu\}$) to be perpendicular one to the other, equal in magnitude, and proportional to ζ^{-1}. The magnitude of Poynting flux, $|S_{(f)}| = |E_{(f)}|^2 = |B_{(f)}|^2$, is

$$\begin{aligned} |S_{(f)}| = \frac{e^2}{(4\pi)^2} \frac{a_o^2 \sin^2\vartheta_+ \csc^4\chi_+}{4(1 + a_o^2\alpha^2\cos^2\vartheta_+)^3} \big[\cos^2\vartheta_* \sin^2\varphi_* \\ + \big(\cos\varphi_* + a_o^{-1}\sqrt{a_o^2 + \alpha^{-2}}\sin\vartheta_* \csc\vartheta_+\big)^2 \big]\, \zeta^{-2} \end{aligned} \tag{22}$$

(see Fig. 5 for the definition of angles χ_+, ϑ_+, ϑ_*, φ_*). These results are typical for a *radiative* field. Most interestingly, this radiative aspect depends on the specific geodesic along which a given point on *spacelike* $\mathcal{I}^+$ is approached (cf. [36]). Moreover, the radiative character does not disappear even for static sources but it does along null geodesics emanating from such sources.

Summarizing, we have analyzed the fields of uniformly accelerated charges in a de Sitter universe which go over to classical Born's fields in

the limit $\Lambda \to 0$. Aside from some similarities found, the generalized fields provide the models showing how a positive cosmological constant implies essential differences from physics in flat spacetime: advanced effects occur inevitably, and the character of the far fields depends substantially on the way in which future (spacelike) infinity is approached.

8. Outlook

One of the best known boost-rotation symmetric spacetimes is the vacuum C-metric, representing two black holes uniformly accelerated in opposite directions due to "strings" located either between them, or extending from each of them to infinity. This solution can be generalized to include charged and rotating black holes, and also a non-zero cosmological constant. The "cosmological" C-metric has attracted some attention not long ago (see, e.g., Refs. [39,40]) but its radiative properties have not been investigated until very recently[41]. Notice that two objects uniformly accelerated in opposite directions in de Sitter-like universe do not approach velocity of light asymptotically and form a permanently bounded system — in contrast to analogous objects in spacetimes with $\Lambda = 0$ (compare Figs. 1 and 3, 4). By a detailed analysis of the asymptotic behavior of the "charged, cosmological" C-metric near future spacelike infinity, the peeling properties have been demonstrated and the dependence of both gravitational and electromagnetic fields on spatial directions from which a point at null infinity is approached exhibited.[41] This interesting effect, typical for spacetimes with a spacelike future infinity, can thus be explicitly seen here for the first time. The radiation pattern at a point of $\mathcal{I}^+$ in the case of electromagnetic fields due to the exact charged C-metric is the same as that from test electromagnetic fields from uniformly accelerated charges in de Sitter space discussed in Sec. 7 (see, e.g., Eq. (22)).

More generally, it can be demonstrated that the dependence of radiative parts of the fields on a direction along which a spacelike $\mathcal{I}^+$ is approached is completely determined by the algebraic (Petrov) type of the fields.[42]

Acknowledgments

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